Fuzzy Adaptive Control of the Highly Nonlinear Heat-Exchanger Plant

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Abstract

In this paper a new fuzzy adaptive cancellation control scheme is presented. The basic part of the fuzzy adaptive cancellation controller is the inverse fuzzy model, which is given in the form of a fuzzy relational matrix. The fuzzy adaptive controller has been evaluated by implementation on heat-exchanger plant, which exhibits non-linear and time-varying behaviour. To realize on-line identification, a recursive fuzzy identification algorithm based on the relational matrix has been developed. This identification algorithm offers very fast convergence of estimated parameters and the advantage of implicitly estimated operating point. It is shown that for mutable processes the adaptive fuzzy cancellation controller is superior to the classical model-reference adaptive control.

1 Introduction

When the process parameters of a controlled process either are poorly known or vary during operation, the use of adaptive control technique is generally necessary to obtain a high-performance control system. Many solutions have been proposed in order to make control systems adaptive. One of those solutions, model-reference adaptive system, evolved in the late 50s. The main innovation of this system is the presence of a reference model which specifies the desired dynamics of the closed-loop system. The reference model can also be implicitly included in the closed-loop system as a cancellation principle. The cancellation principle of model-reference control has been used to develop a fuzzy adaptive system.

In this paper the fuzzy adaptive cancellation controller which is based on the inverse fuzzy relational model is investigated. The adaptive scheme is tested on a real mutable process with single dynamics, i.e. a highly nonlinear heat-exchanger pilot plant whose parameters vary during the operation time. The nature of the process requires an adaptive scheme. The fuzzy adaptive scheme is based on the recursive fuzzy identification of the inverse matrix model. The algorithm requires on-line identification developed and discussed in this paper. The fuzzy relational matrix model is obtained on the basis of the fuzzified process input and output variables. The relational matrix model actually represents the relationship between those fuzzified variables and is presented with the fuzzy relational matrix.

The paper is organized as follows: first the algorithm of fuzzy identification is developed and its recursive form is presented. Also some problems relating to recursive identification are discussed. In section 2 the fuzzy cancellation adaptive controller is developed and discussed. In section 3 the application of adaptive approache on heat-exchanger is presented.

2 Fuzzy Identification

In this section Takagi-Sugeno fuzzy models or systems are discused. Suppose the rule base of a fuzzy system is as follows

$$\boldsymbol{R}_i : \textbf{IF } x_1 \textit{ is } \boldsymbol{A}_i \textit{ and } x_2 \textit{ is } \boldsymbol{B}_i \textbf{ THEN } y = f_i(x_1, x_2) \qquad i = 1, \dots N \tag{1}$$

where x_1 and x_2 are input variables of the fuzzy system, y is an output variable, A_i, B_i are fuzzy sets characterized by their membership functions. The **IF**-parts (antecedents) of the rules describe fuzzy regions in the space of input variables and the **THEN**-parts (consequent) are functions of the inputs, usually defined as

$$f_i(x_1, x_2) = a_i x_i + b_i x_2 + r_i \tag{2}$$

where a_i, b_i are the consequent parameters. For $a_i = b_i = 0$ the model becomes a Takagi-Sugeno fuzzy model of the zeroth order. Such a very simplified fuzzy model can be regarded as a collection of several linear models applied locally in the fuzzy regions defined by the rule antecedents. Smoth transition from one subspace to another is assured by the overlapping of the fuzzy regions.

Fuzzy identification based on fuzzy Sugeno model of the zeroth order is concerned with fuzzy rules of the following form

$$\boldsymbol{R}_i : \textbf{IF } x_1 \textit{ is } \boldsymbol{A}_i \textit{ and } x_2 \textit{ is } \boldsymbol{B}_i \textbf{ THEN } y = r_i \qquad i = 1, \dots N \tag{3}$$

This is a singleton fuzzy model where f_i is a constant.

Rule-premises are formulated as fuzzy AND relations on the cartesian product set $X = X_1 \times X_2$, and several rules are connected by logical OR. Fuzzification of a crisp input value x_1 produces a column fuzzy set

$$\boldsymbol{\mu}(x_1) = \left[\mu_{A_1}(x_1), \mu_{A_2}(x_1), \dots \mu_{A_m}(x_1)\right]^T \tag{4}$$

and similarly for a crisp value x_2. The degrees of fulfillment for all possible AND combinations of rule premises are calculated and written into a matrix $\boldsymbol{S}$. If the algebraic product is used as AND operator, this matrix can be directly obtained by the multiplication:

$$\boldsymbol{S} = \boldsymbol{\mu}_1 \otimes \boldsymbol{\mu}_2^T = \boldsymbol{\mu}_1 \cdot \boldsymbol{\mu}_2^T \tag{5}$$

where $\boldsymbol{\mu}_1(m \times 1)$ and $\boldsymbol{\mu}_2(n \times 1)$ are the input fuzzy sets.

A crisp output value y is computed by simplified algorithm for singletons as a weighted mean value (Center of Singeltons):

$$y = \frac{\sum_{i=1}^{n} \sum_{j=1}^{m} s_{ij} r_{ij}}{\sum_{i=1}^{n} \sum_{j=1}^{m} s_{ij}} \tag{6}$$

The dimension of the matrix $\boldsymbol{S}(m \times n)$, which actually represents the structure of the model, depends on the dimensions of the input fuzzy sets $\boldsymbol{\mu}_1(m \times 1)$ and $\boldsymbol{\mu}_2(n \times 1)$. The fuzzy relational matrix $\boldsymbol{R}$ consists of elements r_{ij}.

In order to apply a standard least-squares method to estimate the parameters r_{ij}, the vectors $\boldsymbol{s}$ and $\boldsymbol{r}$ are formed from $\boldsymbol{S}$ and $\boldsymbol{R}$, respectively

$$\begin{aligned} \boldsymbol{s} &= (s_{11}\ s_{12} \ldots s_{1n} \ldots s_{m1}\ s_{m2} \ldots s_{mn})^T \\ \boldsymbol{r} &= (r_{11}\ r_{12} \ldots r_{1n} \ldots r_{m1}\ r_{m2} \ldots r_{mn})^T \end{aligned} \tag{7}$$

Using these vectors, equation 6 is rewritten as

$$y = \frac{\boldsymbol{s}^T \cdot \boldsymbol{r}}{\boldsymbol{s}^T \cdot \boldsymbol{I}} = \frac{\boldsymbol{s}^T(x_1, x_2) \cdot \boldsymbol{r}}{\boldsymbol{s}^T(x_1, x_2) \cdot \boldsymbol{I}} \tag{8}$$

where $\boldsymbol{I}$ defines the vector of ones of the same dimension $(n \cdot m \times 1)$ as $\boldsymbol{s}$ and $\boldsymbol{r}$. The elements r_{ij} are estimated on the basis of the observations which are obtained in equidistant time intervals by measuring the process input and output. A system of linear equations is constructed from the above equations for the time points $t = t_1, t = t_2, \ldots, t = t_N$

$$\begin{bmatrix} \boldsymbol{s}^T(t_1) \\ \boldsymbol{s}^T(t_2) \\ \vdots \\ \boldsymbol{s}^T(t_N) \end{bmatrix} \cdot \boldsymbol{r} = \begin{bmatrix} \boldsymbol{s}^T(t_1) \cdot \boldsymbol{I} y(t_1) \\ \boldsymbol{s}^T(t_2) \cdot \boldsymbol{I} y(t_2) \\ \vdots \\ \boldsymbol{s}^T(t_N) \cdot \boldsymbol{I} y(t_N) \end{bmatrix} \tag{9}$$

The system is of the form:

$$\boldsymbol{\Psi} \cdot \boldsymbol{r} = \boldsymbol{\Omega} \tag{10}$$

with a known nonsquare matrix $\boldsymbol{\Psi}$ and a known vector $\boldsymbol{\Omega}$. The solution of this overdetermined system is obtained by taking the pseudo-inverse as an optimal solution of vector $\boldsymbol{r}$ in a least squares sense:

$$\boldsymbol{r} = (\boldsymbol{\Psi}^T \boldsymbol{\Psi})^{-1} \boldsymbol{\Psi}^T \boldsymbol{\Omega} \tag{11}$$

where $\boldsymbol{\Psi}$ stands for fuzzified data matrix with dimension $N \times (n \cdot m)$ and $\boldsymbol{\Omega}$ has dimension $N \times 1$.

In the case of more than two input variables (MISO multi-input-single-output fuzzy system), matrices $\boldsymbol{S}$ and $\boldsymbol{R}$ are no longer matrices, but both become a tensor, defined on the total product space of the inputs.

When the observations are obtained sequentially, the recursive equation can be derived. The procedure is known as recursive identification. The acquisition of new data at certain time instants gives information on the current behaviour of the process. Each observation, each pair of input-output data, contributes a new equation which gives a lot of information on the process parameters. The whole algorithm should be calculated in the time between two samples. This restriction can be a serious problem. The identification procedure in recursive form saves some computing time and is suitable when the process parameters are time-varying. When the process parameters are changing during the operation, it is necessary to eliminate the influence of old data. This can be done using a method with exponential weighting. The old data are forgotten exponentially because the new samples give more adequate information on the current behaviour of the process. The method of recursive fuzzy identification with exponential weighting is based on the loss function

$$J(\hat{\mathbf{r}}_c) = \sum_{k=1}^{N} \lambda^{N-k} \left(y(k) - \frac{\boldsymbol{s}^T(k)}{\boldsymbol{s}(k)\boldsymbol{I}} \hat{\boldsymbol{r}}_c(k)\right)^2, \tag{12}$$

where $y(k)$ is the current value of process output, $\boldsymbol{s}^T(k)$ is the fuzzy data vector in transposed form, $\hat{\boldsymbol{r}}_c(k)$ is the current value of the estimated fuzzy relational vector, and λ is the forgetting factor. The proper value of the forgetting factor is chosen between 0.95 and 0.98 as proposed by Isermann, Lachmann and Matko [6].

Optimizing the loss function (12) and expressing the normalized fuzzy data vector as

$$\boldsymbol{s}_n(k) = \frac{\boldsymbol{s}^T(k)}{\boldsymbol{s}(k)\boldsymbol{I}}, \tag{13}$$

the recursive fuzzy identification with exponential weighting is obtained in the following form

$$\hat{\boldsymbol{r}}_c(k+1) = \hat{\boldsymbol{r}}_c(k) + \boldsymbol{K}(k)(y(k+1) - \boldsymbol{s}_n^T(k)\hat{\boldsymbol{r}}_c(k)) \tag{14}$$

$$\boldsymbol{K}(k) = \boldsymbol{P}(k)\boldsymbol{s}_n(k+1)[\lambda + \boldsymbol{s}_n^T(k+1)\boldsymbol{P}(k)\boldsymbol{s}_n(k)]^{-1} \tag{15}$$

$$\boldsymbol{P}(k+1) = \frac{1}{\lambda}[\boldsymbol{I} - \boldsymbol{K}(k)\boldsymbol{s}_n^T(k+1)]\boldsymbol{P}(k). \tag{16}$$

Eq.(14) expresses the new estimate $\hat{\boldsymbol{r}}_c(k+1)$ of the process parameters as a correction of the previous estimate $\hat{\boldsymbol{r}}_c(k)$. The correction is proportional to the error between the observed value of y at time $k+1$ and the prediction of $y(k+1)$ which is based on preceding estimates of parameters.

The initial value of the matrix is chosen as the covariance of initial values of the estimated fuzzy relational vector parameters $\hat{\boldsymbol{r}}_c$ as follows

$$\boldsymbol{P}(0) = cov(\hat{\boldsymbol{r}}_c - \boldsymbol{r}). \tag{17}$$

When there is no a priori information on the initial values of estimated parameters, the initial values of the matrix $\boldsymbol{P}(0)$ have to be chosen sufficiently large

$$\boldsymbol{P}(0) = \alpha \boldsymbol{I}, \quad \alpha >> 1, \tag{18}$$

and the initial values of the estimated fuzzy relational vector parameters are set to

$$\hat{r}_c(0) = \mathbf{0}. \tag{19}$$

The application of recursive fuzzy identification requires continuous monitoring and supervision of several parameters. The identification algorithm can be started in a closed loop after specifying free parameters and setting the initial conditions for parameter estimation. These problems are connected with the start-up procedure or pre-identification. Another problem is the persistent excitation in the closed loop. All these problems are discussed in the section on supervision and coordination.

3 Fuzzy Adaptive Control

Fuzzy models give some advantage in comparison to conventional models, and this justifies the introduction of the fuzzy adaptive system. The fuzzy adaptive system consists of recursive identification of the fuzzy model and the fuzzy controller. In our case, the fuzzy model of the process is given in the form of the relational matrix representation of the process inverse. This model is used by the cancellation fuzzy controller. In the next two subsections, the relational matrix identification and the fuzzy cancellation controller will be given.

3.1 The fuzzy adaptive cancellation controller based on fuzzy relational matrix

The conventional cancellation control is used for tracking control problems and the model-reference control. The basic idea of the cancellation control is to design a controller which ensures the desired closed-loop response. Basically this is the controller which can be encountered in model-reference adaptive control. The closed-loop transfer function, consisting of a conventional cancellation controller and the process, is supposed to be equal to the prescribed reference model

$$G_m(z) = \frac{Y(z)}{W(z)} = \frac{G_r(z)G_p(z)}{1 + G_r(z)G_p(z)}, \tag{20}$$

where $G_m(z)$ is the reference model, $G_r(z)$ is the transfer function of controller in the direct loop and $G_p(z)$ is the transfer function of the process. To obtain the closed-loop response defined with the reference model as in the Eq.(20), the controller transfer function is equal to

$$G_r(z) = \frac{1}{G_p(z)} \cdot \frac{G_m(z)}{1 - G_m(z)}. \tag{21}$$

The goal of cancellation controllers is to cancel the process dynamic with the cancellation part of the controller and to introduce the reference dynamic. Mathematically, this can be described as follows

$$G_{comp} = \frac{1}{G_p(z)}. \tag{22}$$

This equation expresses the cancellation part and the following equation the noncancellation part

$$G_{noncomp}(z) = \frac{G_m(z)}{1 - G_m(z)} = \frac{B_m(z)z^{-d}}{A_m(z) - B_m(z)z^{-d}} \tag{23}$$

where $B_m(z)z^{-d}$ is the numerator and $A_m(z)$ the denominator of the reference-model transfer function.

The cancellation characteristic involves some constraints. The first problem is the realization of a controller connected with the time delay of the process. The transport time delay of the process could not be compensated for and should be incorporated into the closed-loop system as the delay of the reference model. So the delay of the reference-model should be chosen equal to or greater than the delay of the controlled process. A detailed explanation is found in [6] and [7]. Other constraints are related to the cancellation of zeros and poles which lie outside the unit circle. The cancellation implies that all poles and zeros must lie inside the unit circle, and so the cancellation controller in its original form is applicable only to stable minimum-phase processes. This drawback can be eliminated by appropriate measures. The cancellation can be avoided if the desired polynomial $B_m(z)$ includes all zeros of the process outside the unit circle. The polynomial $A_m(z) - B_m(z)z^{-d}$ must include all unstable poles of the process.

The fuzzy cancellation controller is designed with the same considerations as the conventional cancellation controller. It also consists of a cancellation and a noncancellation part. The cancellation part is in the case of fuzzy cancellation controllers realized as the fuzzy inverse model of the process. The fuzzy inverse model of a mutable single dynamic process has two inputs $y(k)$ and $y(k-1)$ and one output $u_m(k)$. The fuzzy inverse model is obtained using the input error model shown in Fig. 1 and described by the fuzzy relational equation

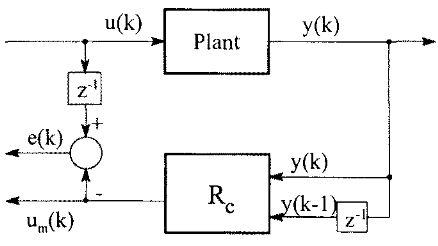

Figure 1: The input error model

$$U_{k+1} = Y_k \circ Y_{k-1} \circ \boldsymbol{R}_c. \tag{24}$$

The noncancellation part of the controller is the same as for the conventional cancellation controller

$$\frac{U_{aux}(z)}{E(z)} = \frac{G_m(z)}{1 - G_m(z)}, \tag{25}$$

where $E(z)$ is the Z-transform of the error signal $e(k)$ and $U_{aux}(z)$ is the Z-transform of auxiliary control signal $u_{aux}(k)$, which is the input of the inverse fuzzy model. The fuzzy data matrix of the controller's inverse model is composed of the actual value of the auxiliary control signal $u_{aux}(k)$ and of the one time sample delayed auxiliary control signal $u_{aux}(k-1)$. The output of the inverse model is the current control signal $u(k)$. Fig. 2 shows the fuzzy cancellation controller in the closed loop.

The relation between the auxiliary control signal and the control signal is described by the following equation

$$u(k) = \frac{\boldsymbol{s}^T(\boldsymbol{\mu}(u_{aux}(k)), \boldsymbol{\mu}(u_{aux}(k-1))) \cdot \hat{\boldsymbol{r}}_c(k)}{\boldsymbol{s}^T(\boldsymbol{\mu}(u_{aux}(k)), \boldsymbol{\mu}(u_{aux}(k-1))) \cdot \boldsymbol{I}}, \tag{26}$$

where $\boldsymbol{s}^T(\boldsymbol{\mu}(u_{aux}(k)), \boldsymbol{\mu}(u_{aux}(k-1)))$ is the fuzzy data vector and $\hat{\boldsymbol{r}}_c$ is the current estimate of the fuzzy relational vector of the inverse process.

The described algorithm of the fuzzy cancellation adaptive controller exhibits some advantages over the conventional adaptive technique. These advantages are based on fuzzy identification which enables the identification of nonlinear process dynamic and also implicitly describes the operating point of the process.

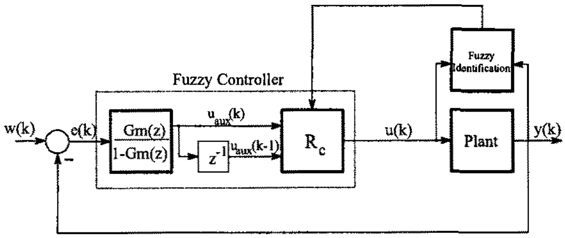

Figure 2: The scheme of the fuzzy cancellation adaptive system

3.2 Supervision and coordination

The implementation of fuzzy adaptive control requires an additional supervision and coordination system to eliminate and avoid all expected or unexpected changes in the operating conditions of the controlled process in the adaptive control loop. These changes may result in unacceptable or unstable control behaviour of the fuzzy adaptive system. Therefore a continuous monitoring and supervision of the fuzzy adaptive control loop functions are required.

A very important part of supervision is the P-controller which controls the difference between the variable on the inverse model input $u_{aux}(k)$ and process output $y(k)$. In the ideal case, both variables should be equal. When the controller action of the implemented fuzzy cancellation controller is too weak or too strong, an additional supervision controller is needed. The input into the P-supervisory controller is the difference between the variables $y(k)$ and $u_{aux}(k)$ and the output is $u_p(k)$, which together with the output from the fuzzy cancellation controller $u_f(k)$ forms the control signal $u(k)$ of the process. Fig. 3 shows the fuzzy cancellation controller with the P-supervisory controller in the closed loop.

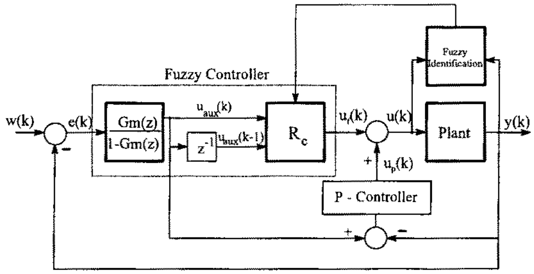

Figure 3: The scheme of the fuzzy cancellation adaptive system with supervision system

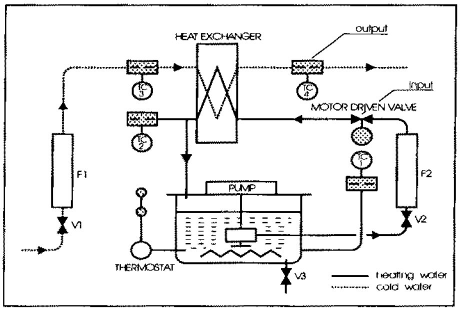

Figure 4: The heat exchanger pilot plant

4 Adaptive Control of the Heat Exchanger Pilot Plant

The adaptive approach discussed has been implemented in a real temperature plant, which consists of a plate heat exchanger, through which hot water from an electrically heated reservoir is continuously circulated in the counter-current flow to cold process fluid (cold water). The thermocouples are located in the inlet and outlet flows of the exchanger; both flow rates can be visually monitored. Power to the heater may be controlled by time proportioning control using the external control loop. The flow of the heating fluid can be controlled by the proportional motor driven valve. A schematic diagram of the plant is shown in Figure 4. The temperature $T_1(z,t)$ is measured on temperature sensor TC4, $T_2(z,t)$ measured on temperature sensor TC1 and $T_s(z,t)$ represent the temperatures of the cold water, heating water and the iron wall respectively, $v_1(t)$ measured on flow sensor F1 and $v_2(t)$ measured on flow sensor F2, are velocities of cold and heating water, k_1 and k_2 constants which include the heat transfer coefficients and the physical dimensions of the heat exchanger.

The input of heat-exchanger model is the current velocity of the heating water $v_2(t)$ and outlet temperature of the cold water T_1 is defined as the output. In order to obtain the simple model of the heat exchanger, theoretical modelling would be very difficult because of the unknown nonlinear behaviour. Furthermore, the heat exchanger is just one part of the plant, so the sensors and the actuators should also be modelled. The motor driven valve exhibit a strong nonlinear and time-varying behaviour.

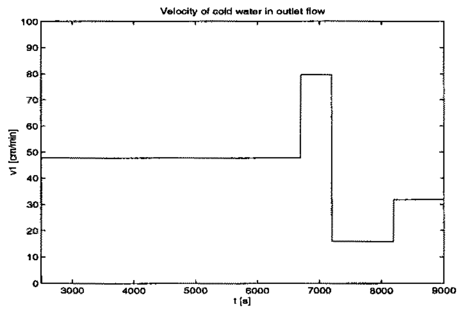

Figure 5: Velocity of cold water in outlet flow

System modelling based on conservational laws and first principles would be a very difficult, expensive and time consuming task. Instead, fuzzy identification of the process is used. Although the process is very complex, it could be presented as a model with approximately first order dynamics with small time delay, with significantly time varying parameters and nonlinearities according to the operating point.

During the experiments, some values of the physical parameters (velocity v_1 of the outlet and the temperature T_2 at the inlet of the exchanger) which are supposed to be constant were changing. The time course of the changing outlet flow v_1 is presented in Figure 5. The inlet temperature of the heated water T_2 is controled using a simple on-off controller and vary between 60^0C and 65^0C. These variations have a great influence on the gain and on the dominant time constant of the process. The period of the first 2500s has been used for the pre-identification in the closed-loop. In this periode the robust PI-controller has been used to control the inlet temperature T_2. Afterwards, the fuzzy adaptive controller was switched on. The output of the closed-loop $y(t)$ and the reference model output $y_m(t)$ are shown in Figure 6. In this Figure only the periode of fuzzy adaptive contol is shown. In spite of very changing condition (changing outlet velocity v_1 and inlet temperature T_1) and nonlinear system dynamics, the closed-loop response obtained using proposed fuzzy adaptive algorithm exhibits a very good performance in both modes, in model following mode and disturbance rejection mode.

5 Conclusion

In this paper the fuzzy adaptive cancellation control scheme is presented. The development of a new fuzzy adaptive scheme was motivated by the unsatisfactory results obtained using conventional adaptive techniques. Regarding the real-time experiments realized on the heat-exchanger plant, it can be seen that the novel algorithm introduces faster convergence and better performance in the presence of nonlinearity and unmeasured dynamics. The main disadvantages of this algorithm are stability analysis, which is still unsolved, and the very complicated relational matrix in the case of a higher order system. The

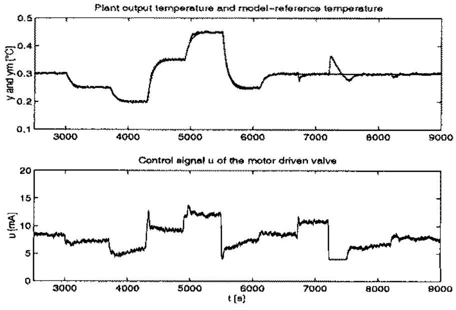

Figure 6: Plant output temperature and model-reference temperature

proposed approach seems to be usable for time-varying or nonlinear systems with simple dynamics. In such cases, the proposed algorithm gives some advantages over other conventional model-reference adaptive techniques.

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